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NATURAL CONVECTION FROM ISOTHERMAL FLAT SURFACES

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Abstract—Local heat-transfer coefficients along a flat plate in natural convection in air were measured using Boelter–Schmidt type heat flux meters. Experiments were carried out for different temperature differences in heating and cooling, and with inclinations varying from the horizontal "facing upwards" position, through the vertical position, to the horizontal "facing downwards" position.

The results are presented in terms of local Nusselt number as a function of the local Grashof number "tangential component". All runs were in the range accepted as that of laminar boundary layer flow. However, under certain conditions when the normal velocity component of the air is directed away from the surface, separated flow is indicated along the trailing part of the surface, well before turbulence sets in in the boundary layer. Separation starts at a certain point along the surface. This point is nearer to the leading edge the higher the temperature difference, and the larger the inclination of the surface to the vertical.

In a separation region, the flux density is uniform. In all other regions the results agreed closely with established theories of laminar boundary layer flow.

A leading adiabatic section, used in some of the experiments, did not affect the results.

An appendix gives relations recommended for engineering calculations.

NOMENCLATURE

c,	specific heat of air [kcal/kg C];
C_{Pr},	a factor that is a function of N_{Pr} [dimensionless];
g,	gravitational acceleration [m/h^2];
h,	heat transfer coefficient [kcal/h m^2 C];
k,	thermal conductivity of air [kcal/h m C];
K,	a constant [dimensionless];
L,	height of surface [m];
m,	index [dimensionless];
N_{Gr},	$g\beta\theta L^3/\nu^2$, average Grashof number for length L [dimensionless];
$N_{Gr,x}$,	$g\beta\theta x^3/\nu^2$, local Grashof number at distance x [dimensionless];
N_{Nu},	hL/k, average Nusselt number for length L [dimensionless];
$N_{Nu,x}$,	hx/k, local Nusselt number at distance x [dimensionless];
N_{Pr},	$\mu c/k$, Prandtl number [dimensionless];
N_{Ra},	$N_{Pr} . N_{Gr}$, average Rayleigh number [dimensionless];
$N_{Ra,x}$,	$N_{Pr} . N_{Gr,x}$, local Rayleigh number [dimensionless];
q'',	heat flux density [kcal/h m^2];
x,	distance along surface measured from leading edge [m];
α,	angle of inclination of surface to the vertical [degrees]: positive if the fluid's normal velocity component is directed away from surface;
β,	coefficient of volumetric expansion of air [C^{-1}];
θ,	temperature difference between surface and air [C];
μ,	dynamic viscosity of air [kg/m h];
ν,	kinematic viscosity of air [m^2/h].

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INTRODUCTION

THE PROBLEM of natural convection is one of the oldest problems in heat transfer. Most of the work on this subject is on vertical surfaces and horizontal cylinders. Except for a few papers on the stability of the flow, all the work is on the determination of either the average heat transfer coefficient, or on the velocity and temperature distributions in the boundary layer.

Contrary to this, the data on natural convection from horizontal surfaces is scarce. Indeed, a large part of Jakob's [1] discussion on the subject is based on experimental results on boiling from horizontal surfaces. The remaining part of the discussion quotes results of Griffiths and Davis in 1922, and of Weise in 1935. Fishenden and Saunders [2] recommend certain relations without giving a reference. All other references seem to either quote one of these results, or neglect the subject completely, and there does not seem to be any more recent publications on this subject.

As to natural convection from inclined flat surfaces, there seems to be two papers only on the subject, one of them published after the completion of the present work. In these two papers, the inclination is limited to 45 degrees from the vertical, with conditions of laminar boundary-layer flow.

The above shows the lack of sufficient data on natural convection from inclined surfaces. The present work was designed to fill this gap by the experimental determination of the local heat-transfer coefficients along flat surfaces at all inclinations.

PREVIOUS WORK

In this part, the more important publications that bear directly on the present work are briefly reviewed.

The mathematical set-up of the problem of natural convection was first given by Nusselt [3] as the conservation equations of mass, momentum and energy. He used these relations to derive the dimensionless groups pertinent to the problem which are now known as Nusselt (N_{Nu}), Grashof (N_{Gr}) and Prandtl (N_{Pr}) numbers. Consequently, natural convection relations, whether local or average, could be put in the following general form

$$N_{Nu} = N_{Nu}(N_{Pr}, N_{Gr}). \tag{1}$$

In particular, when the inertia forces could be neglected in comparison with buoyancy and frictional forces, the above relation could be put as

$$N_{Nu} = N_{Nu}(N_{Pr} \cdot N_{Gr}) = N_{Nu}(N_{Ra}). \tag{2}$$

This is usually put in the form

$$N_{Nu} = \bar{K} N_{Ra}^{m}. \tag{3}$$

For local values, a similar equation is used with the same index, namely

$$N_{Nu} = K N_{Ra,x}^{m}. \tag{4}$$

Later, Schmidt and Beckmann [4] studied experimentally the temperature and velocity distributions adjacent to vertical 125 and 500 mm high plates in natural convection. Complementing these experiments, they simplified the form of the governing equations by omitting certain terms, particularly by assuming that the velocity component normal to the plate is negligible compared to the tangential component. These simplifying assumptions made the analysis less valid in the low Grashof number range. In the same work [4], E. Pohlhausen reduced the simplified partial differential equations to ordinary ones by applying similarity transformations. A numerical solution of these equations was carried out for $N_{Pr} = 0{\cdot}733$ and compared with the experimental results. The calculated temperature distribution agreed closely with the experimental measurements, whereas significant differences between the calculated and measured velocity fields were found.

With the general form known, Squire [5] used a general third-order relation to describe the velocity distribution in the boundary layer, and a second-order relation to represent the temperature distribution in the same layer. Substituting these into the integrated momentum and energy equations, he obtained an expression of the local Nusselt number in terms of Prandtl number and the local Grashof number. With more generalization* and giving more details, Eckert [6] obtained the same expression, namely

$$N_{Nu} = 0{\cdot}508\,[N_{Pr}N_{Gr,x}/(0{\cdot}952 + N_{Pr})]^{\frac{1}{4}}. \quad (5)$$

Ostrach [7] solved the simplified ordinary differential equations of Schmidt and Beckmann by computer for the following values of Prandtl numbers: 0·01, 0·72, 0·733, 1, 2, 10, 100 and 1000. He compared his results with those obtained from equation (5) and equation (4) with the index $m = \frac{1}{4}$ and the constant $K = 0{\cdot}411$ cited by McAdams [8]. He found better agreement with equation (5) than with equation (4), although the latter agreed better with experimental results in the high Prandtl number range.

Ostrach's (as well as Pohlhausen's) results give the local Nusselt number along a vertical plate as

$$N_{Nu,x} = C_{Pr}\,N_{Gr,x}^{\frac{1}{4}}. \quad (6)$$

The factor C_{Pr} is a function of Prandtl number as follows:

Table 1.

N_{Pr}	0·7	0·72	0·733	1
C_{Pr}	0·353*	0·357	0·359	0·401

* Extrapolated value.

Rich [9] seems to be the first to consider the case of inclined plates, both analytically and experimentally. In the simplified equation of Schmidt and Beckmann, he replaced the gravitational acceleration g by its tangential component $g \cos \alpha$ parallel to the surface. He carried out his experiments on a plate 400 mm long and 100 mm wide. Using an inferior quality interferometer with corrections, he determined the temperature field for distances up to 325 mm, and a local Grashof number range of 10^6–10^9. He experimented with inclinations α to the vertical of 0, 10, 20, 30 and 40 degrees. Experiments were not carried out beyond this inclination because a velocity component in the third dimension became significant.

Rich's measured temperatures in the boundary layer were somewhat lower than predicted by analysis, while the local heat transfer coefficients were in good agreement with equation (5) modified by using $N_{Gr,x} \cos \alpha$ instead of $N_{Gr,x}$. In general, the results show that the effects of the terms neglected in the analysis were not significant in the range of the experiments.

Pohlhausen's "boundary layer solution" was improved by Yang and Jerger [10] who used it as a zeroth order approximation together with a first order perturbation. The perturbation equations needed a new set of boundary conditions in which velocities at the plate and "boundary layer" edges are assumed. Computer solutions are given for $N_{Pr} = 0{\cdot}72$ and 10. The results correlate the velocity field measurements better, but do not much improve the zeroth order approximation of the temperature distribution, being already in good agreement with the experimental measurements of Schmidt and Beckmann. However, the perturbation method gives slightly lower Nusselt numbers than the zeroth order approximation.

Brodowicz [11] studied experimentally the flow and temperature fields around a vertical hot plate 240 mm high. He found that these fields did not extend far upstream of the leading edge, although he used an insulated starting section in some of this experiments. Neither did the wake at the upper edge affect the upstream field appreciably. This is in general agreement with the boundary conditions used by Yang and Jerger. However, Brodowicz found that the flow at the plate edges and at the boundary layer edge is not steady.

Following the same procedure, Kierkus [12]

* By using the coefficient of volumetric expansion β instead of the reciprocal of the absolute temperature

extended the method of Yang and Jerger to inclined plates. He included a term neglected by Rich. This term is unsymmetrical about $\alpha = 0$ and, therefore, distinguishes between upward- and downward-faces positions of the plate. He carried out experiments similar to those of Brodowicz on a square 250×250 mm plate for inclinations $\alpha = 0, \pm 15, \pm 30$ and ± 45 degrees. The flow was "completely laminar, except for the region near the trailing edge ... 'above' the plate". Kierkus' conclusions are the same as those of Yang and Jerger in the sense that the first order perturbation correction for the temperature profile is hardly noticeable.

APPARATUS

The apparatus used in the experiments is fully described in [13] and is shown schematically in Fig. 1 for the vertical hot plate position. To the left is the heat transfer surface of the cover plate (A) which is 200 mm wide and 504 mm

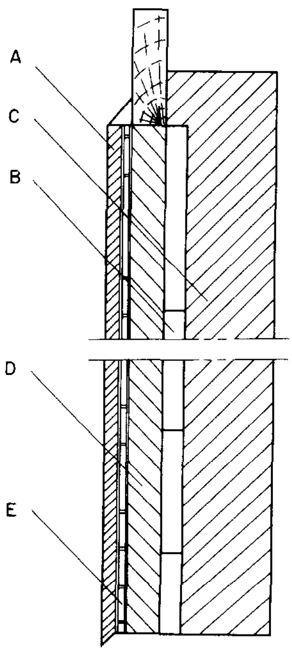

FIG. 1. Sketch of apparatus.

high, and dissipates the heat flux to the surrounding air.

The heat flux emanates from a 5 mm thick layer of four heat exchangers (B) to which hot or cold water of controlled temperature and rate is supplied from an Ultrathermostat. Each heat exchanger is divided into a number of 30 mm wide passes. High water flow rates were used to limit the water temperature variation in a heat exchanger to about 0·6 C. The heat exchangers are insulated at the back by a 70 mm thick layer of cotton wool (C).

The heat flux leaving the heat exchangers passes through an 8 mm thick electrolytic copper plate (D) to maintain an isothermal surface before the heat flux meters layer (E). Eight heat flux meters of the Boelter–Schmidt type [1] are used. Each meter is made of 200 turns of 0·12 mm dia. constantan wire wound around the central 80 mm of a 180×19 mm bakelite strip, 1·5 mm thick. The wires are copper plated to half the width of the strip. The spaces between the heat flux meters are filled by strips of the same bakelite sheet. The whole layer is varnished and sandwiched between two thin paper covers to improve the thermal contact, and ensure proper electric insulation.

The heat flux meters are used to measure the local heat flux densities at 17, 36, 55, 103, 153, 230, 332 and 475 mm from the leading edge of the cover plate (A). The sensitivities of the meters used ranged from 0·0723 to 0·0815 mV/(kcal h^{-1} m^{-2}) with a maximum deviation of less than ± 5 per cent. Reproducibility tests carried out months later increased this maximum deviation for some of the meters to within $+9$ per cent of the original mean. The cover plate is a 4 mm thick brass plate, nickel plated on the exposed surface to minimize the radiant heat flux. It is fitted with 24 copper–constantan thermocouples to measure directly the temperature difference between the heat-transfer surface and the air. Three thermocouples at 60 mm intervals are fitted to the heat transfer surface along the centre line of each heat flux meter. The leading edge of the cover plate is chamfered

.o a sharp edge and could be fitted, flush with it, with wooden adiabatic leading sections of different lengths.

The whole assembly could be rotated about a long horizontal pin. Fixation is provided for the vertical position of the heat transfer surface, or any inclined position every 15 degrees through a full circle.

CONDITION OF EXPERIMENT

The experiments were carried out in a closed, thick-walled, large room free of air currents. Six runs were made with six different temperature differences, as follows:

Table 2.

Temperature difference (C)	Air temperature (C)
$-8{\cdot}05 \pm 0{\cdot}2$	$28{\cdot}4 \pm 0{\cdot}8$*
$7{\cdot}6 \pm 0{\cdot}2$	$28{\cdot}3 \pm 0{\cdot}6$
$15{\cdot}0 \pm 0{\cdot}4$	$28{\cdot}0 \pm 0{\cdot}8$
$22{\cdot}4 \pm 0{\cdot}6$	$28{\cdot}6 \pm 0{\cdot}6$
$30{\cdot}5 \pm 0{\cdot}8$	$27{\cdot}0 \pm 0{\cdot}4$
$38{\cdot}2 \pm 1{\cdot}0$	$27{\cdot}2 \pm 0{\cdot}6$

* The air dew-point temperature limited the cold surface experiments to this single run.

In each run, the surface inclination angle α was varied from $+90$ to -90 degrees from the vertical, every 15 degrees. For inclination, a positive angle indicates a hot plate facing upwards or a cold plate facing downwards, a negative angle indicates the opposite cases. In all hot surface experiments the sharp leading edge was lowermost, whereas it was uppermost for cold surface experiments.

The temperature difference between the surface and air was measured directly by the thermocouples. The construction of the apparatus allowed a close control of this temperature difference. During the experiments, the temperature deviation along the surface centre line was within ± 1 per cent of the temperature difference between the surface and air. For all the surface, this deviation was within ± 3 per cent [13]. The error due to this deviation is negligibly small.

Some of the experiments with hot surface were carried out with an adiabatic, 200 mm long, leading section.

MEASUREMENTS

The results obtained directly are essentially those of 'total' flux density distribution at different temperature differences and surface inclinations. The total flux densities read include the radiant flux densities; they were, therefore, corrected for radiation. This correction amounted to about 10 per cent or less of the total heat flux density, according to the angle of inclination [13].

Two different classes of results were obtained, according to whether the angles of inclination are positive or negative. These two classes are represented in Figs. 2 and 3, each of which represents a half run. Figure 2 shows the flux density distribution on a cold plate facing upwards (negative inclination). The maximum flux density is near the leading (uppermost) edge and decreases along the plate, except for horizontal plates where the flux density increases again at the other end. At a given locality, smaller angles of inclination give higher flux densities; the maximum being, therefore, for the vertical surface. This figure, Fig. 2, is typical for all negative inclinations irrespective of whether it is a case of heating or cooling, and independent of the temperature difference which affects magnitude only [13].

Figure 3 shows the flux density distribution for a hot surface of positive inclination (facing upwards). The dashed parts of the curves show anticipated distributions where experimental points are lacking. Most of the general trends of Fig. 2 are present also in this case. However, as the inclination from the vertical increases, a sudden jump in the flux density takes place near the trailing edge. The point at which this jump takes place moves forward as the inclination increases. After a short distance from the jump,

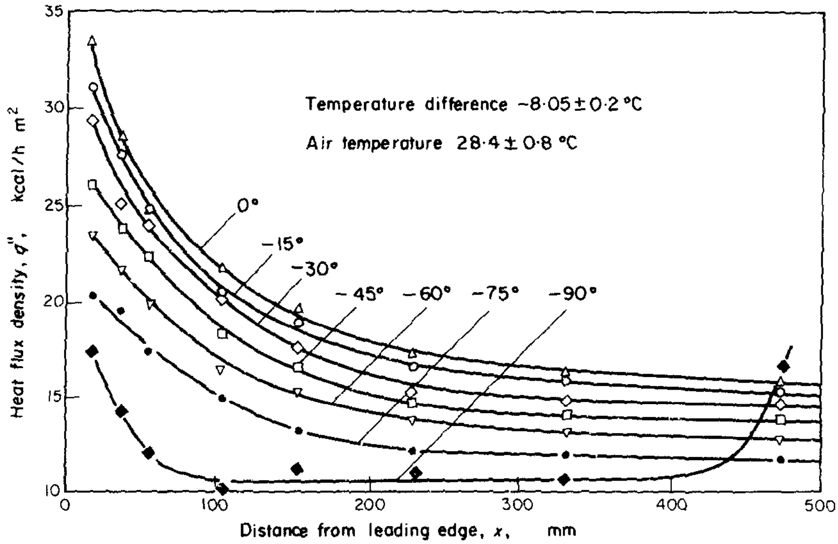

FIG. 2. Natural convection local heat flux density distribution for surface at different negative inclinations in air (Cold surface, $\theta = -8{\cdot}05$ deg C)

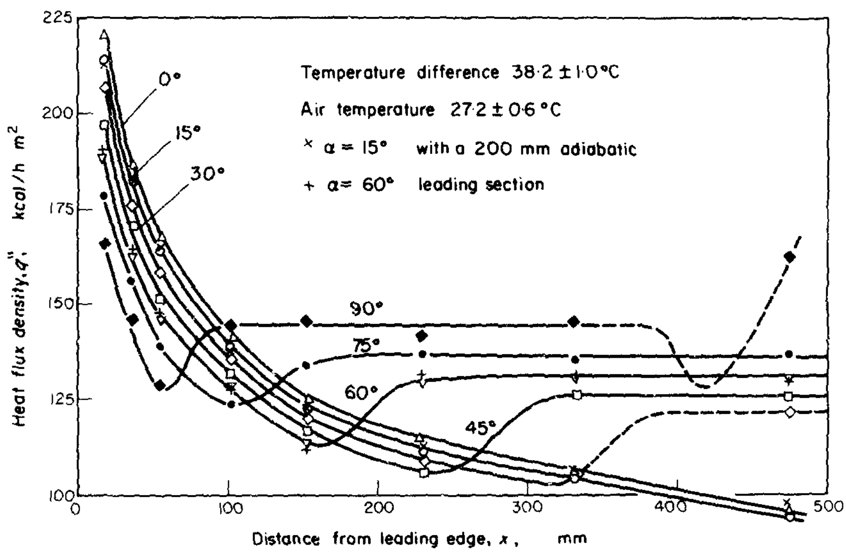

FIG. 3. Natural convection local heat flux density distribution for surface at different positive inclinations in air (Hot surface, $\theta = 38{\cdot}2$ deg C)

the flux density remains practically uniform. This trend continues till the horizontal position where the flux density in the middle part of the surface is the highest for all inclinations; however, a dip in the flux density occurs near the edge and increases again to its highest local value at the edge itself.

Some of the experimental points shown in Fig. 3 are taken with a 200 mm long adiabatic leading section. They are indicated by crosses and do not differ appreciably from those taken without the leading section. The insignificant effect of the starting section support the findings of Brodowicz [11], which were published after the completion of this work.

The trend shown in Fig. 3 is typical for positive inclinations, except for the positions of the sudden jumps. For a given inclination, the point at which the jump takes place moves forward as the temperature difference increases [13]. This could be seen from Fig. 4 which is plotted

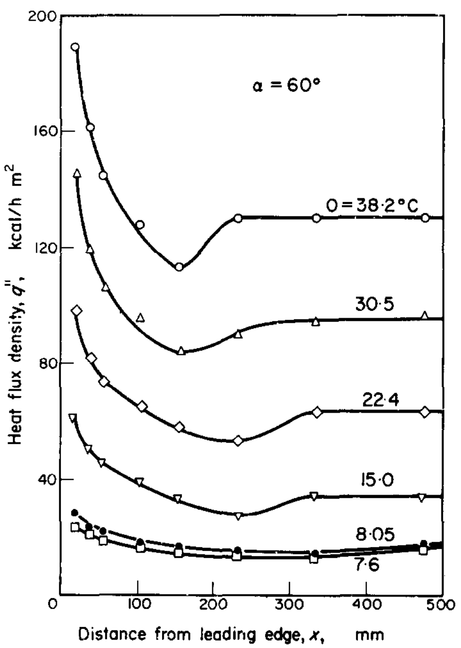

FIG. 4. Natural convection local heat flux density distribution for hot surface of inclination $\alpha = 60$ degrees in air for different temperature differences.

for an inclination of 60 degrees and various temperature differences.

This gradual recession of the laminar boundary layer region, shows that the phenomenon indicated by a jump is the same for all inclinations, including the horizontal position. Since the jump indicates separation for the horizontal position,* it also indicates separation for other inclinations. Indeed, this is supported by the fact that a jump is absent when the inclination is negative, including the horizontal (-90 degrees) position. Had a jump been an indication of turbulence in the boundary layer, it would have appeared for negative inclinations also where the flow is constrained by the surface.

Separation was detected by Rich [9] as "flow in the third dimension" for inclinations of more than 40 degrees, but was not defined as separation because of the interferometric method he used to determine the flux. Kierkus [12] detected disturbance near the trailing edge "above" his plate, which was probably too short to establish this as separation.

Separation is, conceivably, due to the effect of the velocity component normal to the surface. The above results show that separation sets in on surfaces of positive inclinations well before turbulence does.

On the other hand, for horizontal ($+90$ degrees) surfaces, the flux distribution near the edge and its similarity to the cases of inclined surfaces, indicates the predominance of the velocity component tangential to the surface at its edges.

RESULTS

The results are compiled in terms of the usual dimensionless groups, namely the local Nusselt number, $N_{Nu,x} = hx/k$, and the local Grashof number, $N_{Gr,x} = g\beta\theta x^3/\nu^2$. However, in the present case where inclined surfaces are considered, the buoyancy acceleration component along the surface $g\beta\theta \cos \alpha$ gives the body force in this direction. Consequently,

* See, for example, Mikheyev [14].

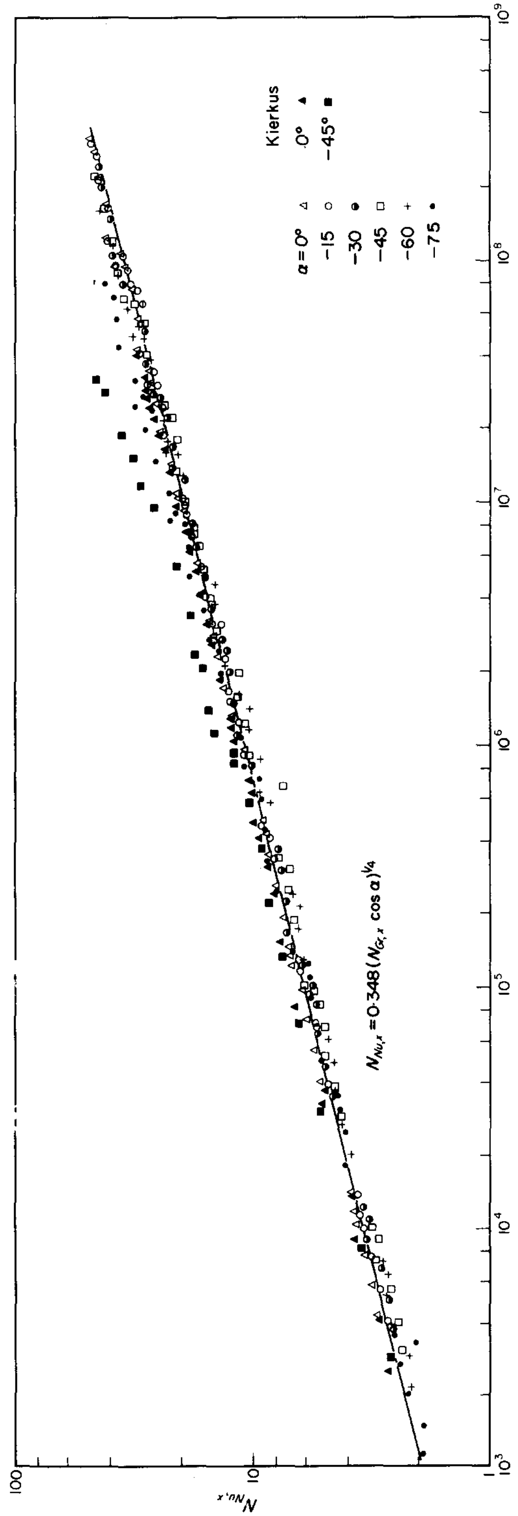

FIG. 5. Natural convection from negatively inclined flat surfaces in air.

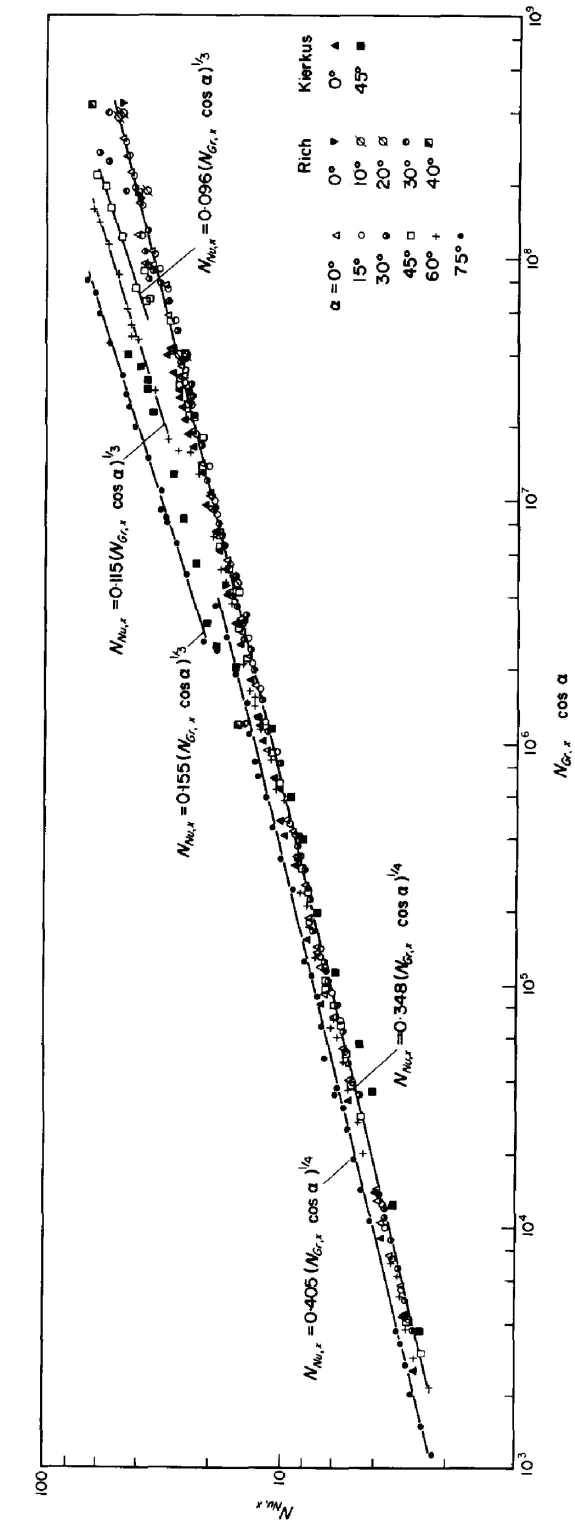

FIG. 6. Natural convection from positively inclined flat surfaces in air.

a local Grashof number "tangential component" $N_{Gr,x} \cos \alpha$ is used. It should be noted that the effect of Prandtl number, being an intensive property of the fluid, could not be detected by the present experiments because a single fluid (air) was used.

In calculating the dimensionless groups, the fluid properties are arbitrarily taken at the surface temperature, and the distance x measured from the leading edge. The local heat transfer coefficient h is determined from the local flux density q'' and the *mean* temperature difference θ for the whole surface.

Because of the different trends detected, the results are represented graphically in two separate figures. Figure 5 is a compilation of experimental data for negative angles of inclination, whereas Fig. 6 compiles data for positive inclinations. The two plots are given on logarithmic scales with ordinates $N_{Nu,x}$ as function of $N_{Gr,x} \cos \alpha$ on the abscissae. Each of the two figures covers a range of local Grashof number tangential component between $1 \cdot 1 \times 10^3$–3×10^8 which is well within the limit of laminar boundary layer flow of about $N_{Gr,x} < 2 \times 10^9$ as given by Fishenden and Saunders [2] and by Cheeswright [15] for vertical plates. Each figure compiles about 300 experimental points for both hot and cold surfaces that include the vertical position, but exclude the horizontal positions. The different surface inclinations are denoted by separate points in the two figures without distinction between the heating and the cooling cases.

Because of its simpler form, the set of negative inclination angles represented by Fig. 5 is discussed first. As could be seen from this figure, the experimental points converge well along a line of inclination $\frac{1}{4}$. The points for $\alpha = -75$ degrees, and $N_{Gr,x} \cos \alpha > 5 \times 10^6$ deviate from this general line. Indeed, except for these points and five other points, the bulk of the data could be correlated with a maximum deviation of less than ± 10 per cent by the following relation:

$$N_{Nu,x} = 0 \cdot 348 \, (N_{Gr,x} \cos \alpha)^{\frac{1}{4}}. \tag{7}$$

This is only 3 per cent less than that predicted by Pohlhausen [4] and Ostrach [7] for vertical plates. The deviation is in the direction predicted by Yang and Jerger [10] for vertical plates, and by Kierkus [12]. To this figure, the experimental points of Kierkus [12] are added. They are in excellent agreement with the present results in the range $N_{Gr,x} \cos \alpha < 10^6$, whereas considerable differences appear beyond this range. The reason for these differences is not clear; it may be due to instabilities near the trailing edge of short plates operating with large temperature differences.

Figure 6 compiles the experimental points for positive inclination angles. As could be seen from this figure, equation (7) above represents a large bulk of the data, even better than it does in Fig. 5. Equation (7) represents the following cases with a maximum deviation of about ± 5 per cent in the following ranges of $N_{Gr,x} \cos \alpha$.

Table 3.

α (degrees)	$N_{Gr,x} \cos \alpha$ range
0	$<3 \times 10^8$
15	$<3 \times 10^8$
30	$<2 \times 10^8$
45	$<6 \times 10^7$
60	$<3 \times 10^6$

For angles of inclination higher than 60 degrees, the local Nusselt numbers tend to have higher values than those given by equation (7), as could be seen from Fig. 6. This tendency, probably due to the appreciable velocity component normal to the surface, starts at $\alpha = 60$ degrees, the experimental points for which are slightly, but consistently, higher than the experimental points for lower inclinations. For $\alpha = 75$ degrees, the experimental points are distinctly higher than the bulk of the points for other inclinations. For this inclination and the range $1 \cdot 2 \times 10^3 < N_{Gr,x} \cos \alpha < 2 \times 10^6$, the experimental points are correlated by

$$N_{Nu,x} = 0 \cdot 405 \, (N_{Gr,x} \cos \alpha)^{\frac{1}{4}}. \tag{8}$$

The line that represents this relation is also shown in Fig. 6.

Beyond the ranges given in Table 3, to the end of the range, the experimental points deviate from the trends given above due to the "jumps" shown in Figs. 3 and 4. The experimental points in these high tangential Grashof number ranges could be correlated by relations of the form:

$$N_{Nu,x} = K_\alpha (N_{Gr,x} \cos \alpha)^{\frac{1}{3}}. \qquad (9)$$

The value of the constant K_α depends on the surface inclination according to the following table:

Table 4.

α (degrees)	45	60	75
K_α (dimensionless)	0·096	0·115	0·155

Equation (9) with the appropriate values of K_α from Table 4 represents the experimental points, except these in limited transition regions, with a maximum deviation of about ± 5 per cent also.

The ranges of applicability of equation (9) are necessarily those of separation. The upper limit for these ranges is certainly not restricted by the range of the present experiments. Indeed equation (9) should be applicable to any point on positively inclined surfaces beyond the ranges indicated by Table 3.

The index $\frac{1}{3}$ of equation (9) shows that the heat transfer coefficient is independent of position along the plate as indicated by distance x. This fact is shown graphically in Figs. 3 and 4; and in itself, is an indication of separation. Indeed, recent experiments by Coutanceau [16] on vertical plates give a higher index of 0·41* when the flow is turbulent. As mentioned before, this is a condition that would not be reached for positive inclinations unless the inclination is very small.

* Coutanceau used the dimensionless group (gx^3/ν^2) instead of the local Grashof number in his correlation.

Experimental points of Rich [9] and Kierkus [12] are shown also in Fig. 6. All of Rich's points, and those of Kierkus in the range of $N_{Gr,x} \cos \alpha < 3 \times 10^6$, are in excellent agreement with the present results. Beyond this range, Kierkus' points indicate higher values of the local Nusselt number. This may be due to the same reason mentioned above in connection with Fig. 5 for negatively inclined surfaces.

Considering a horizontal surface, the case is different from that of inclined ones because the flow is three dimensional. As mentioned before, tangential velocity components occur near the surface edges. This shows the importance of the surface's aspect ratio. General relations could not, therefore, be obtained from the present experiments. However, taking the central part of the plate to represent natural convection from very large plates where the edge effects could be neglected, the following relations are recommended.

For hot plates facing upwards and cold plates facing downwards ($\alpha = +90$ degrees), the following equation represents the experimental data to within ± 15 per cent.

$$N_{Nu} = 0{\cdot}12\, N_{Gr}^{\frac{1}{3}}. \qquad (10)$$

For this case, Fishenden and Saunders [2] recommend a relation of the form of equation (3) with an index $m = \frac{1}{3}$. When a value of 0·7 is used for the Prandtl number of air, this relation gives values that are higher by less than 4 per cent of those given by equation (10).

For hot plates facing downwards and cold plates facing upwards ($\alpha = -90$ degrees), the following relation represents the experimental data to within ± 10 per cent

$$N_{Nu} = 0{\cdot}06\, N_{Gr}^{\frac{1}{3}}. \qquad (11)$$

The relation recommended by Fishenden and Saunders for this case uses an index of $\frac{1}{4}$, which increases to less than $\frac{1}{3}$ at high values of Rayleigh number. According to the reasoning used here, this would be mainly applicable to fairly narrow plates where laminar flow prevails near the edges. However, Jakob [1] cites the results of Griffiths and Davis for the heat flux in this case as 50 per cent of the heat flux from vertical plates under similar conditions. In comparison, equation (11) gives values that are 55–58 per cent of the average for the corresponding vertical surface case.

CONCLUSIONS

Experiments were carried out to determine the local heat transfer coefficients along an inclined flat surface in natural convection. They showed that along the surface, and well before the boundary layer turns to turbulence, separation of flow takes place along the trailing part at positive angles of inclination. In other localities, and at small inclinations, the results agree closely with those predicted by modern analyses of laminar boundary-layer flow.

An adiabatic leading section, used in some of the experiments, did not affect the results significantly.

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APPENDIX

Engineering Relations

The results of the present experiments may be extended to other fluids by using the form of equation (4) with a value of 0·7 for the Prandtl number of air. Further, integration of such an equation along the length of the plate gives an equation of the form of equation (3) in which the average Nusselt number, N_{Nu}, based on the total length L of the plate, is a function of the Rayleigh number tangential component, $N_{Ra} \cos \alpha$, based also on the plate height. Integration of equation (4) for $0 \leqslant x \leqslant L$ gives

$$N_{Nu} = (K/3m)(N_{Ra} \cos \alpha)^m. \tag{12}$$

Laminar flow prevails on the leading sections of inclined plates in the regions given in Table 5. It should be noted that in this table, the limit set for the $-60 < \alpha < +15$ degrees positions is that for the start of turbulence on vertical surfaces [2, 15]. The limit given for the +30 degrees position is the upper limit for the present experiments where there is evidence, though not conclusive, of the onset of separation. For the large inclinations, separation sets in at the limits given as determined by the present experiments. For the conditions

Table 5.

α (degrees)	$N_{Ra} \cos \alpha$
-75	$\leqslant 5{\cdot}5 \times 10^6$
-60 to $+15$	$\leqslant 2{\cdot}2 \times 10^9$
$+30$	$\leqslant 2{\cdot}2 \times 10^8$
$+45$	$\leqslant 6{\cdot}5 \times 10^7$
$+60$	$\leqslant 3.3 \times 10^6$

of Table 5, equation (7) gives

$$N_{Nu} = 0.507\,(N_{Ra}\cos\alpha)^{\frac{1}{4}}. \qquad (13)$$

For an inclination of +75 degrees and $N_{Ra}\cos\alpha \leqslant 2.2 \times 10^6$, equation (8) is applicable and gives

$$N_{Nu} = 0.59\,(N_{Ra}\cos\alpha)^{\frac{1}{4}}. \qquad (14)$$

For surface regions downstream of the above values, the flow is either turbulent or separated. In either case, two flow regimes exist along the surface, each with its own constant K and index m. In such a case, integration along the two parts of the plate gives an equation of the following form

$$N_{Nu} = (K_1/3m_1)\,(N_{Ra,M}\cos\alpha)^{m_1} + (K_2/3m_2)\,(N_{Ra}^{m_2} - N_{Ra,M}^{m_2})\,(\cos\alpha)^{m_2}.$$

In this relation the Rayleigh number $N_{Ra,M} < N_{Ra}$ is the intermediate value at which the flow changes form. Its value may be taken as that given in Table 5. The above relation could be put in the following general form

$$N_{Nu} = K_4 + K_5(N_{Ra}^{m_2} - K_6)\,(\cos\alpha)^{m_2}. \qquad (15)$$

For the negatively inclined surfaces, separation is not expected and the flow would change to turbulence at a certain critical value of $N_{Ra}\cos\alpha$. However, data on turbulence is available for vertical surfaces only, for which case Eckert and Jackson give a relation quoted by Warner in [15] as

$$N_{Nu} = 0.021\,N_{Ra}^{0.4}. \qquad (16)$$

It is not unreasonable to extend this relation to negatively inclined plates, and plates of slightly positive inclinations. This could be done by using $N_{Ra}\cos\alpha$ instead of N_{Ra} in equation (16). Since the nature of disturbance detected along a plate with $\alpha = -75$ degrees is not clear at present, it is assumed that equation (16) is applicable for inclinations of -60 to $+15$ degrees only. It is conceivable that an unknown upper limit of $N_{Ra}\cos\alpha$ for the applicability of this equation exists for positively inclined plates beyond which the flow is separated. For the range of inclinations mentioned, the following form of equation (15) is recommended

$$N_{Nu} = 110 + 0.021\,(N_{Ra}^{0.4} - 6150)\,(\cos\alpha)^{0.4}. \qquad (17)$$

For positively inclined plates, separation sets in before turbulence and should continue for all values of $N_{Ra}\cos\alpha$ higher than those given by Table 5. Consequently, equation (9), rather than equation (16), is used to determine the constants of equation (14) for positive inclinations. This gives an index $m_2 = \frac{1}{3}$, and the following constants

Table 6.

α (degrees)	K_4	K_5	K_6
+30	62	0·1	600
+45	45·5	0·11	400
+60	21·6	0·13	150

For large horizontal plates where the edge effects could be neglected, equations 10 and 11 yield the following relations

For $\alpha = +90$ degrees

$$N_{Nu} = 0.135\,N_{Ra}^{\frac{1}{3}}. \qquad (18)$$

For $\alpha = -90$ degrees

$$N_{Nu} = 0.068\,N_{Ra}^{\frac{1}{3}}. \qquad (19)$$

CONVECTION NATURELLE PAR DES SURFACES PLANES ISOTHERMES

Résumé—Les coefficients de transfert thermique local le long d'une plaque plane pour une convection naturelle dans l'air ont été mesurés à l'aide de fluxmètres du type Boelter–Schmidt. Des expériences ont été réalisées pour plusieurs différences de température au cours d'échauffement et de refroidissement, avec des inclinaisons variant depuis la position horizontale, face orientée vers le haut, à la position horizontale, face orientée vers le bas, en passant par la verticale.

Les résultats sont présentés à l'aide du nombre de Nusselt local fonction du nombre de Grashof local. Tous les essais ont été conduits dans le cas d'un écoulement à couche limite laminaire. Dans certaines conditions, où la composante principale de vitesse de l'air est dirigée vers l'extérieur de la surface, l'écoulement séparé est observé bien avant l'apparition de la turbulence dans la couche limite. La région où se fait la séparation est d'autant plus près du bord d'attaque que la différence de température est grande et que l'inclinaison de la surface par rapport à la verticale est forte.

Dans une région de séparation, la densité de flux est uniforme. Dans toutes les autres régions les résultats sont en très bon accord avec les théories de l'écoulement laminaire de la couche limite.

Une section amont adiabatique, utilisée dans quelques expériences n'a pas affecté les résultats.

En appendice sont données des relations recommandées pour les calculs d'engineering.

NATÜRLICHE KONVEKTION VON ISOTHERMEN EBENEN FLÄCHEN

Zusammenfassung—Mit einem Wärmestrommessgerät vom Typ Schmidt–Boelter wurden die örtlichen Wärmestromkoeffizienten entlang einer ebenen Platte bei natürlicher Konvektion in Luft ermittelt. Die Experimente wurden ausgeführt für verschiedene Temperaturdifferenzen bei Aufheizen und Abkühlen und für verschiedene Plattenneigungen von der horizontalen Stellung mit nach oben gerichteter Heizfläche über die vertikale bis zur horizontalen Stellung mit nach unten gerichteter Heizfläche.

Die Ergebnisse werden dargestellt in Ausdrücken für örtliche Nusseltzahl als Funktion der "tangentialen Komponente" der örtlichen Grashofzahl. Alle Messreihen lagen in einem Bereich, für den laminare Grenzschichtströmung angenommen wurde. Unter bestimmten Bedingungen jedoch tritt, wenn die zur Platte senkrechte Geschwindigkeitskomponente der Luft von der Oberfläche weggerichtet war, im hinteren Teil der Oberfläche Ablösung auf, noch ehe in der Grenzschicht Turbulenz einsetzt. Die Ablösung beginnt an einem bestimmten Punkt der Oberfläche, der um so näher an der Anlaufkante liegt, je höher die Temperaturdifferenz und je grösser die Neigung der Oberfläche zur Senkrechten ist.

Im Ablösungsgebiet war die Wärmestromdichte einheitlich. In allen anderen Gebieten stimmen die Ergebnisse gut mit vorhandenen Theorien über laminare Grenzschichtströmung überein.

Ein adiabater Abschnitt im Anlaufgebiet, der in manchen Versuchen verwendet wurde, blieb ohne Einfluss auf die Ergebnisse. Im Anhang werden Beziehungen für Ingenieurberechnungen empfohlen.

ЕСТЕСТВЕННАЯ КОНВЕКЦИЯ НА ИЗОТЕРМИЧЕСКОЙ ПЛОСКОЙ ПОВЕРХНОСТИ

Аннотация—С помощью датчиков типа Больтера-Шмидта измерялись локальные коэффициенты теплообмена вдоль плоской пластины при естественной конвекции в воздухе. Эксперименты проводились при различных перепадах температур при нагреве и охлаждении с различными положениями пластины: горизонтальном («лицом вверх»), вертикальном и горизонтальном («лицом вниз»).

Результаты представлены в виде зависимости числа Нуссельта от «тангенциальной составляющей» числа Грасгофа. Все опыты проводились в режиме ламинарного пограничного слоя. Однако, при определенных условиях, когда нормальная составляющая скорости воздуха направлена от поверхности, отрыв наблюдается в кормовой области поверхности намного раньше возникновения турбулентности в пограничном слое. Отрыв возникает в определенной точке на поверхности. Эта точка расположена тем ближе к передней кромке, чем больше разность температур и чем больше относительно вертикали наклон поверхности.

В области отрыва плотность потока одинакова. Во всех других областях результаты хорошо согласуются с известными теориями ламинарного пограничного слоя.

В некоторых экспериментах с адиабатической передней кромкой она не оказывала влияния на результаты.

В приложении приводятся соотношения, рекомендуемые для инженерных расчетов.